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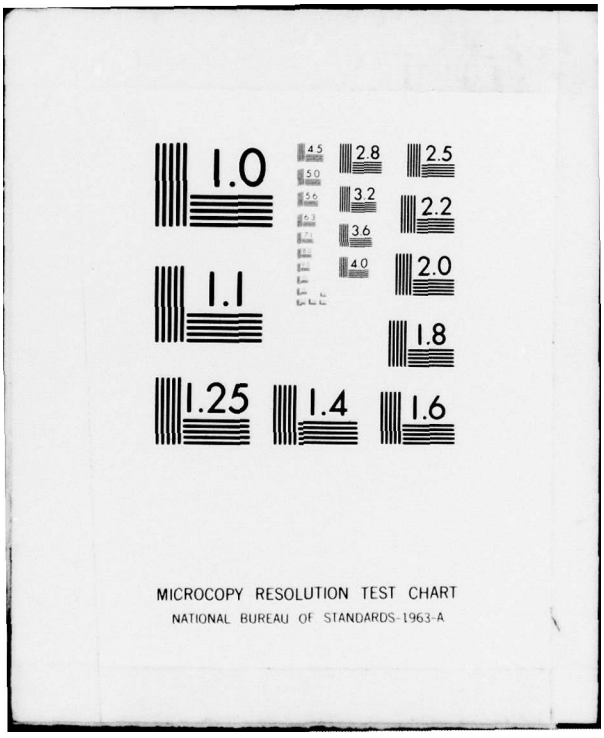
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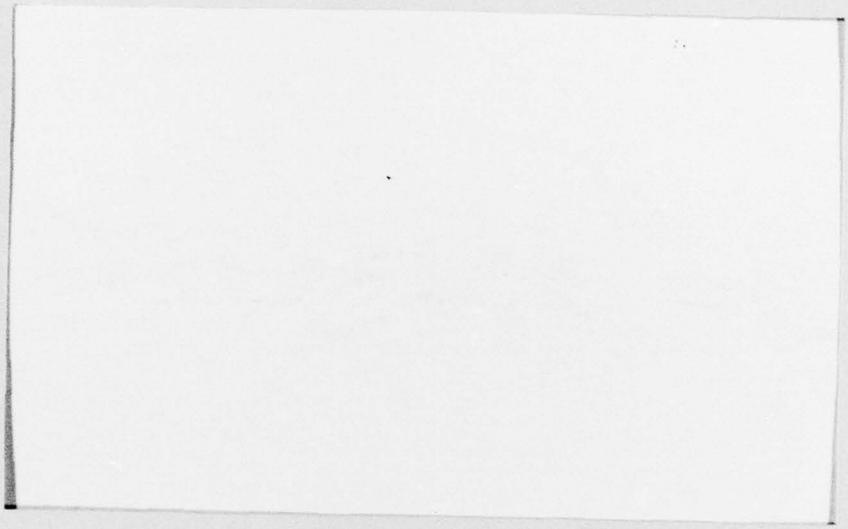

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A TWO-DIMENSIONAL COMPRESSIBLE, NONLINEAR STABILITY ANALYSIS WITH APPLICATIONS TO THE STUDY OF BOUNDARY LAYER TRANSITION

Gabriel Miller & Andrew Callegari

NYU/DAS 79-16

OCTOBER 1979

FINAL REPORT

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A computer program which can investigate the nonlinear response of a compressible boundary layer on a flat plate to any imposed external disturbance has been produced. The program first computes the boundary condition near the leading edge utilizing linear stability theory and then determines the disturbance field downstream as a function of time, utilizing the MacCormack time splitting explicit technique.

The program originally created to provide a fundamental understanding

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of boundary layer response to acoustical disturbances has been modified so that a wide range of parameters can be varied. The effects of Mach number and wall conditions can be computed utilizing the program.

ACKNOWLEDGEMENT

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A TWO-DIMENSIONAL COMPRESSIBLE, NONLINEAR STABILITY ANALYSIS WITH APPLICATIONS TO THE STUDY OF BOUNDARY LAYER TRANSITION

I. INTRODUCTION

Recent analytical efforts to gain a more comprehensive understanding of boundary layer stability have centered on (1) the coupling between the principle linear wave (the Tollmien-Schlichting wave) and its principle harmonics (for example, [1]) and (2) on the development of computer codes to study the nonlinear effects of disturbances propagating through an incompressible boundary layer field (for example, Refs.2, 3, 4). Such investigations extended the earlier incompressible and compressible linear stability analyses (for example, Refs. 5-9) which analyzed the linear stability problem both from spacial and temporal growth processes.

The recent efforts have been motivated by analysis[10] and experiments[11,12,13] which indicated that the process of disturbance growth and instability leading to transition could be subdivided into distinct regions,where first linear, and then nonlinear and three dimensional effects predominate.

Two dimensional nonlinear codes can thus be extremely important tools if one is to properly stimulate the three dimensional bursting phenomenon of transition to turbulence, since such codes can provide the initial conditions for such analysis (similar to the effect of the linear codes which provide the initial conditions for the nonlinear analyses).

While the nonlinear codes available represent a significant advance in computational capability, they have thus far been restricted to incompressible flow calculations. Such programs are unable to analyze the effects of

Mach number, wall temperature, imposed disturbance propagation velocity, etc.

Experiments[14, 15] have indicated that imposed acoustical disturbances effect transition in a low speed flow in a manner similar to free stream turbulence, even though the propagation velocity of the disturbances are as much as two orders of magnitude greater. The frequency range which effects transition at a given Reynolds number is similar to the range associated with freestream turbulence, even though there is a mismatch in freestream propagation velocity (and thus in freestream wave number).

The purpose of the study at New York University for the past three years was to ascertain the mechanisms which cause acoustical disturbances to effect transition in this manner and to develop a computer code which includes both nonlinear effects and the effects of compressibility. The unsteady, two-dimensional, compressible second order boundary layer equations (retaining terms of the order of the reciprocal of Reynolds number squared) have been computed numerically[16]. The system, consistent with the Navier Stokes equations, have been solved by MacCormack explicit scheme with time splitting[17]. In Section II the equations utilized and the method of analysis is developed. Results utilizing the program are presented in Section III. A description of the program's capabilities can be found in Section IV.

II. METHOD OF ANALYSIS

A. Equations of Motion

The equations of motion, the unsteady, compressible Navier Stokes equations, were written in terms of the nondimensional parameters consistent with the boundary layer equations (but terms of order of the reciprocal of the Reynolds number squared were retained). The equations were transformed by a stretching parameter η which packs most points in the boundary layer but allows for a significant distribution of data in the freestream. The stretching utilized was;

$$\eta = 1 - \exp\left(-\alpha\bar{y}/\bar{\delta}^*\right)$$

where $\bar{\delta}^* = \frac{\delta_o^*}{L}\sqrt{Re_L}$, $\bar{y} = \frac{y}{L}\sqrt{Re_L}$, and $\delta o^*(x)$ is the initial boundary layer thickness distribution (the distribution prior to the initiation of perturbed flow). Re_L is the freestream Reynolds number based on characteristic length L (for example, plate length).

The resulting equation system[16], are rewritten here:

$$\frac{\partial\bar{\rho}}{\partial\bar{t}} + \frac{\partial}{\partial\bar{x}}(\bar{\rho}\bar{u}) + \frac{\partial}{\partial\eta}(\bar{\rho}\bar{u})\frac{\partial\eta}{\partial\bar{x}} + (1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial\bar{\rho}\bar{v}}{\partial\eta} = 0 \tag{1}$$

$$\frac{\partial}{\partial\bar{t}}(\bar{\rho}\bar{u}) + \frac{\partial}{\partial\bar{x}}(\bar{\rho}\bar{u}^2 + \bar{p}) + \frac{\partial}{\partial\eta}(\bar{\rho}\bar{u}^2 + \bar{p})\frac{\partial\eta}{\partial\bar{x}} + (1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial}{\partial\eta}(\bar{\rho}\bar{u}\bar{v})$$

$$= \frac{\alpha^2(1-\eta)}{\bar{\delta}^{*2}}\frac{\partial}{\partial\eta}\left[\bar{\mu}(1-\eta)\frac{\partial\bar{u}}{\partial\eta}\right] + \phi_4 \tag{2}$$

$$\frac{\partial}{\partial\bar{t}}(\bar{\rho}\bar{v}) + \frac{\partial}{\partial\bar{x}}(\bar{\rho}\bar{u}\bar{v}) + \frac{\partial}{\partial\eta}(\bar{\rho}\bar{u}\bar{v})\frac{\partial\eta}{\partial\bar{x}} - (1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial}{\partial\eta}(\bar{\rho}\bar{v}^2 + \bar{p}\ Re_L) \tag{3}$$

$$= \frac{4}{3}\frac{\alpha^2}{\bar{\delta}^{*2}}(1-\eta)\frac{\partial}{\partial\eta}\left[\bar{\mu}(1-\eta)\frac{\partial\bar{v}}{\partial\eta}\right] - \frac{2}{3}(1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial}{\partial\eta}\left[\bar{\mu}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{\mu}\frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\eta}\right]$$

$$+ \frac{\partial}{\partial\bar{x}}\left[\bar{\mu}(1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial\bar{u}}{\partial\eta}\right] + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial}{\partial\eta}\left[\bar{\mu}(1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial\bar{u}}{\partial\eta}\right] + \phi_5$$

$$\frac{\partial}{\partial t}(\bar{\rho}\bar{E}) + \frac{\partial}{\partial\bar{x}}(\bar{\rho}\bar{u}\bar{H}) + \frac{\partial}{\partial\eta}(\bar{\rho}\bar{u}\bar{H})\frac{\partial\eta}{\partial\bar{x}} + (1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial}{\partial\eta}(\bar{\rho}\bar{u}\bar{H}) \tag{4}$$

$$= \frac{\alpha^2(1-\eta)}{\bar{\delta}^{*2}}\frac{\partial}{\partial\eta}\left[\bar{\mu}(1-\eta)\frac{\partial\bar{H}}{\partial\eta}\right] + \phi_6$$

$$\bar{p} = \bar{\rho}\bar{T} \tag{5}$$

where

$$\bar{x} = \frac{x}{L};\quad \bar{y} = \frac{y}{L}\cdot\sqrt{Re_L};\quad \bar{t} = t\,\frac{u_\infty}{L}$$

$$\bar{u} = \frac{u}{u_\infty};\quad \bar{v} = \frac{v}{u_\infty}\cdot\sqrt{Re_L};\quad \bar{p} = \frac{p}{\rho_\infty u_\infty^2};\quad \bar{\rho} = \frac{\rho}{\rho_\infty}$$

$$\bar{H} = \frac{H}{u_\infty^2} = \frac{C_pT}{u_\infty^2} + \frac{u^2+v^2}{2u_\infty^2};\quad \bar{E} = \frac{E}{u_\infty^2} = \frac{C_vT}{u_\infty^2} + \frac{u^2+v^2}{2u_\infty^2}$$

$$\bar{\mu} = \frac{\mu}{\mu_\infty} = \frac{T_\infty + 198.6}{T + 198.6}\left(\frac{T}{T_\infty}\right)^{3/2};\quad \bar{T} = \frac{RT}{u_\infty^2} \qquad Re_L = \frac{\rho_\infty u_\infty L}{\mu_\infty}$$

and

$$\begin{aligned}
\phi_4 = \frac{1}{Re_L}\Big\{ & \frac{4}{3}\frac{\partial}{\partial\bar{x}}\left(\bar{\mu}\frac{\partial\bar{u}}{\partial\bar{x}}\right) - \frac{2}{3}\frac{\alpha}{\bar{\delta}^*}\frac{\partial}{\partial\bar{x}}\left[\bar{\mu}(1-\eta)\frac{\partial\bar{v}}{\partial\eta}\right] \\
& + \frac{4}{3}\frac{\partial\eta}{\partial\bar{x}}\frac{\partial}{\partial\eta}\left(\bar{\mu}\frac{\partial\bar{u}}{\partial\bar{x}}\right) - \frac{2}{3}\frac{\alpha}{\bar{\delta}^*}\frac{\partial\eta}{\partial\bar{x}}\frac{\partial}{\partial\eta}\left[\bar{\mu}(1-\eta)\frac{\partial\bar{v}}{\partial\eta}\right] \\
& + (1-\eta)\frac{\alpha}{\bar{\delta}^*}\frac{\partial}{\partial\eta}\left(\bar{\mu}\frac{\partial\bar{v}}{\partial\bar{x}}\right) \\
& + \frac{4}{3}\frac{\partial}{\partial\bar{x}}\left(\bar{\mu}\frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\eta}\right) + \frac{4}{3}\frac{\partial\eta}{\partial\bar{x}}\frac{\partial}{\partial\eta}\left(\bar{\mu}\frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\eta}\right) \\
& + \frac{\alpha}{\bar{\delta}^*}(1-\eta)\frac{\partial}{\partial\eta}\left(\bar{\mu}\frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{v}}{\partial\eta}\right)\Big\}
\end{aligned}$$

$$\begin{aligned}
\phi_5 = \frac{1}{Re_L}\Big[& \frac{\partial}{\partial\bar{x}}\left(\bar{\mu}\frac{\partial\bar{v}}{\partial\bar{x}}\right) + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial}{\partial\eta}\left(\bar{\mu}\frac{\partial\bar{v}}{\partial\bar{x}}\right) \\
& + \frac{\partial}{\partial\bar{x}}\left(\frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{v}}{\partial\eta}\right) + \left(\frac{\partial\eta}{\partial\bar{x}}\right)\frac{\partial}{\partial\eta}\left(\frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{v}}{\partial\eta}\right)\Big]
\end{aligned}$$

$$\begin{aligned}
\phi_6 = \frac{1}{Re_L}\Big\{ & \left[\frac{\partial}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial}{\partial\eta}\right]\left[\bar{\mu}\frac{\partial\bar{H}}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{H}}{\partial\eta}\right] \\
& + \frac{1}{3}\bar{\mu}\left(\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\eta}\right)^2 + \frac{1}{3}\bar{\mu}\left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{v}}{\partial\eta}\right)^2 \\
& - \frac{2}{3}\frac{\alpha(1-\eta)}{\bar{\delta}^*}\bar{\mu}\left(\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\eta}\right)\frac{\partial\bar{v}}{\partial\eta} \\
& + \frac{2\alpha(1-\eta)}{\bar{\delta}^*}\bar{\mu}\left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{v}}{\partial\eta}\right)\frac{\partial\bar{v}}{\partial\eta} \\
& - \frac{1}{2}\left(\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\eta}\right)\left(\frac{\partial(\bar{u}^2)}{\partial\bar{x}} + \frac{\partial\eta}{\partial\bar{x}}\frac{\partial\bar{u}^2}{\partial\eta}\right)
\end{aligned}$$

Equations (1) - (4) have been solved by a MacCormack predictor-corrector explicit scheme with time splitting[17]. At $t = o$, a disturbance field, calculated utilizing linear stability theory (see below), is initiated near the leading edge of a plate. The time dependent flowfield is then calculated numerically from the unsteady, compressible equations, consistent with the imposed initial and boundary conditions.

B. Initial and Boundary Conditions

The investigation has centered on a study of the amplification or dampling of acoustical disturbances propagated into a boundary layer. The experiments of Spangler and Wells[14] who utilized an air-driven, rotating vane sound generator to create the disturbance without producing any appreciable turbulence, were simulated. Both the frequency and the intensity of the sound source had been varied experimentally. In the experiments, a low velocity boundary layer channel was run at a unit Reynolds number of 2.4×10^5/ft., the channel wall representing the flat plate. Measurements of transition occurred at distances on the order of 10 - 20 feet, and thus the length Reynolds numbers of interest were in the 10^6 to 10^7 range.

In order to model this problems, a set of initial and boundary conditions must be established, consistent with the propagation of acoustical disturbances and at the same time consistent with the set of differential equations utilized. The initial data, consistent with equations (1) through (5) under steady state conditions, was originally established utilizing numerical techniques for subsonic boundary layer analysis with normal pressure gradients established by the principal investigator and reported previously.[16] Thus the solution $\bar{t} = o$ was the solution of the two dimensional steady boundary layer equations with normal momentum equation included.

Consistent with Refs. 2 and 3 similar solutions for the initial profiles were studied. Since no problems were encountered, the final program now utilizes such solutions for the initial field.

At the wall ($\eta = o$), for a rigid body, one can neglect the effect of the wave on temperature, and for an adiabatic wall one can establish the following relations;

at $\eta = 0$ $(\bar{x} > \bar{x}^*)$

$$\bar{u} = \bar{v} \qquad \frac{\partial \bar{T}}{\partial \eta} = 0$$

$$\frac{\partial}{\partial \eta}(\ell n \bar{p}) = \frac{\partial}{\partial \eta}[(\ell n \bar{p})]_{\bar{t}=0} \exp\left[-Re_L \int_o^{\bar{t}} \frac{\bar{p}}{\bar{\mu}}\, dt\right]$$

$$\frac{\partial}{\partial \eta}(\ell n \bar{p}) = \frac{\partial}{\partial \eta}[(\ell n \bar{p}]_{\bar{t}=0} \exp\left[-Re_L \int_o^{\bar{t}} \frac{\bar{p}}{\bar{\mu}}\, dt\right]$$

The outer boundary condition ($\eta \to 1$) is established by allowing the wave to travel as a plane wave of arbitrary speed (for the acoustic wave, the speed of sound C_∞).

Much effort was expended on the determination of a proper downstream boundary condition. It was found, consistent with the results of Fasel[2], that the boundary condition that yields the least upstream influence, and is thus superior to other possible ones (including a non-reflective condition) is:

$$\partial^2 \frac{\bar{u}'(\bar{x}_f, \bar{y}, \bar{t})}{\partial \bar{x}^2} = \bar{\alpha}^2 \bar{u}'$$

where $\bar{\alpha} = fL/V_{ph}$ (f is the frequency and V_{ph} the phase velocity), $\bar{u}'$ is the perturbation quantity $(\bar{u} - \bar{u}_o)$, and $\bar{x}_f$ is the downstream boundary. This condition says that at the downstream boundary, the disturbance has a periodic form. At $\bar{t} = o$, a disturbance is initiated

at $\bar{x} \approx 0$ so that the velocity field at $x = \bar{x}^* \approx 0$ and $\bar{t} \geq 0$ can be written as

$$\bar{u}(\bar{x}^*, \bar{y}, \bar{t}) = \bar{u}_0(\bar{x}^*, \bar{y}) + f_1(\bar{y}) \sin(\bar{\omega}\bar{t})$$

where $\bar{u}_0(\bar{x}^*, \bar{y})$ is the velocity profile at $\bar{t} = 0$, $\bar{\omega} = 2\pi fL/U_\infty$ (f being the frequency of the imposed disturbance, in cycles per second) and $f_1(\bar{y})$ is the disturbance profile near the leading edge.

The form of $f_1(\bar{y})$ can in general be prescribed and then the eigenfunction associated with the eigenvalue problem detailed below can be superposed to reconstruct $f_1(\bar{y})$. The complete determination of these eigenfunctions is quite complicated in general depending on the initial boundary layer flow, and leads to a significant numerical problem in linear stability theory. Such complications as the existence of a continuous spectrum of eigenvalues, the presence of a singularity in the linearized eigenvalue problem, and the existence of viscous-type eigenstates which behave singularly in the inviscid limit. [Refs. 18-20] are indicative of studies of such problems.

The other boundary conditions at $x = x^*$ are consistent with linear theory and are to be determined once the form of $f_1(\bar{y})$ is known.

In order to derive appropriate perturbation profiles for use as the initial boundary condition in our program (at $\bar{x} = \bar{x}^*$) the compressible analogue of the Orr-Sommerfeld equations must be solved. For purposes of the present study, we have only considered these equations in the inviscid limit and under the additional assumption of no temperature gradient in the mean flow.

Thus, nondimensionalizing all lengths by the boundary layer thickness δ at a suitable station along the plate, all velocities by the free stream speed of sound, and the pressure by $\rho_\infty c_\infty^2$, the linearized perturbation system considered is;

$$i(ku_o - \omega)\rho + iku\rho_o + \frac{d}{dy}(\rho_o V) = 0 \tag{6}$$

$$i(ku_o - \omega)u + u_o' V + \frac{ik}{\rho_o} p = o \tag{7}$$

$$i(ku_o - \omega)V + \frac{1}{\rho_o}\frac{dp}{dy} = o \tag{8}$$

$$i(ku_o - \omega)T + T_o' V = i\left(\frac{\gamma - 1}{\rho_o}\right)(ku_o - \omega)p \tag{9}$$

$$p/p_o = \rho/\rho_o + T/T_o \tag{10}$$

where bars have been dropped and subscript zero refers to the profile at $t = o$. The appropriate boundary conditions are discussed below. As is customary the normal mode type decomposition has been used, and hence we note that ρ, u, V, p, T are functions of y, the distance normal to the plate. The parameters ω and k, the dimensionless frequency and wave number respectively are given by

$$\omega = \Omega\delta/c_\infty, \quad k = \bar{k}\delta$$

where Ω and $\bar{k}$ are dimensional quantities. The mean flow quantities

$$u_o = M_\infty u_B \text{ and } u_o' = 5.6\left(M_\infty \frac{du_B}{d\eta}\right)$$

are the dimensionless velocity and velocity gradient respectively where M_B is the freestream Mach number, u_B is the dimensionless (with respect to U_∞) Blasius profile and the factor 5.6 results from changing the normal coordinate from η to y.

Several simplifications can be made in equations (6 -10). We have solved for ρ, u, v and T in terms of the pressure p and find that p must satisfy the second order equation

$$p'' - \frac{2ku_o'}{u_o k - \omega} p' + \left[\frac{(u_o k - \omega)^2}{T_o} - k^2\right] p = o \tag{11}$$

we note that the second condition is a normalization condition.

Equations (11) - (12) now provide an eigenvalue problem for the eigenvalue k. Once the possible values of k are determined, the profiles $\bar{x} = \bar{x}^*$ (i.e. the $f_1(\bar{y})$) can be determined and thus the initial and boundary conditions for the calculation are entirely prescribed. To treat this eigenvalue problem a program developed by Mack Ref. (23) has been used. The program has the capability of treating cases where the phase velocity of the disturbance equals the base flow velocity (critical layer) by deformation of the integration contour into the complex plane.

III. RESULTS UTILIZING PROGRAM

The nonlinear computer program for the solution of equations (1) - (4) was first run for cases corresponding to the Reynolds number and frequency range of the experiments of Reference 14. Figure (1) presents the results of three such computations for the forced response of the boundary layer to acoustic waves of different frequencies at a freestream R.M.S. level of .3%. The results are in line with Tollmien Schlicting amplication rates.

An investigation was initiated to determine what the propagation speed inside the boundary layer was. In all cases, while the disturbances were propagating with the speed of sound along the freestream, the waves within the boundary layer were propagating at a speed of the order of the freestream speed, and thus indeed, the waves are the classical Tollmien Schlichting waves. Such a result is presented in Figure 2.

What is essentially occurring is that the wave propagating along the outer edge has little effect on the boundary layer development. Instead, the major effect is the profile at the leading edge, and thus, the effect of the acoustic wave is only to set up the initial disturbance field (i.e., near $x = o$), as described previously. The effect of acoustic waves on transition is, thus similar to the effect of other imposed perturbations such as freestream turbulence.

Another numerical experiment was initiated to test this result. The program has the capability to admit freestream disturbances of any speed. Waves were propagated at the freestream velocity, instead of at the acoustic speed, and all other parameters were held constant. The dashed line in Figure 2 indicates that the effect on the boundary layer ($y/\delta < 1$) is insignificant, and the outer boundary condition only effects the flow-field through its amplitude.

The boundary layer response to an imposed disturbance is thus determined completely once the initial profile (from linear stability theory), is found through a complete eigen-value search. Nonlinear programs should therefore be coupled to linear stability programs which determine the linear (upstream boundary condition) profiles deduced from the multitude of eigen values which have been found to exist.

IV. PROGRAM CAPABILITIES

A computer program which can investigate the nonlinear response of a compressible boundary layer on a flat plate to any imposed external disturbance has been produced. The program first computes the boundary condition near the leading edge utilizing linear stability theory and then determines the disturbance field downstream as a function of time, utilizing the MacCormack time splitting explicit technique.

The program originally created to provide a fundamental understanding of boundary layer response to acoustical disturbances has been modified so that a wide range of parameters can be varied. The effects of Mach number and wall conditions can be computed utilizing the program.

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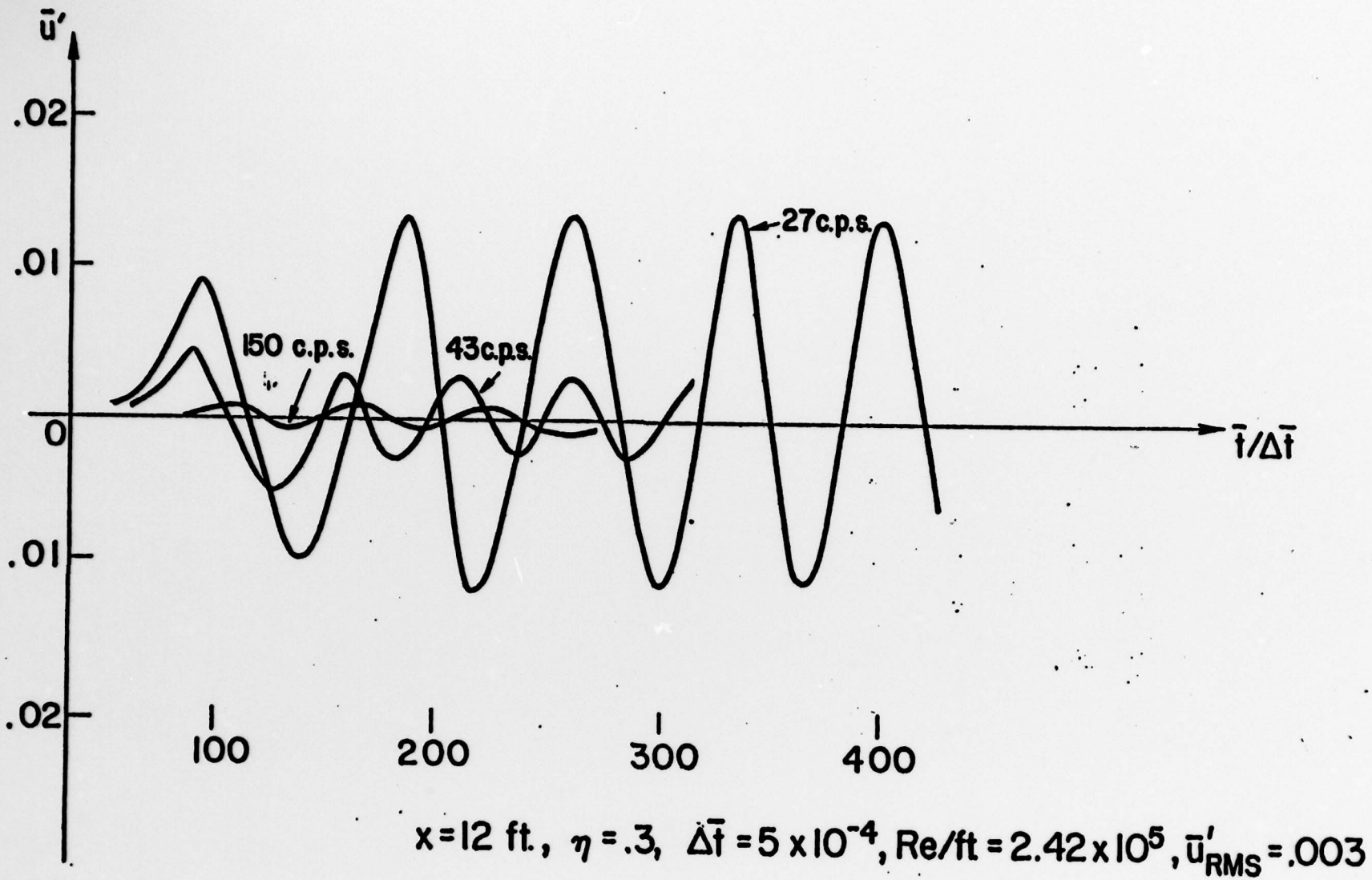

FIGURE 1. FREQUENCY DEPENDENCE ON DAMPING OR AMPLIFICATION OF ACOUSTICAL DISTURBANCE

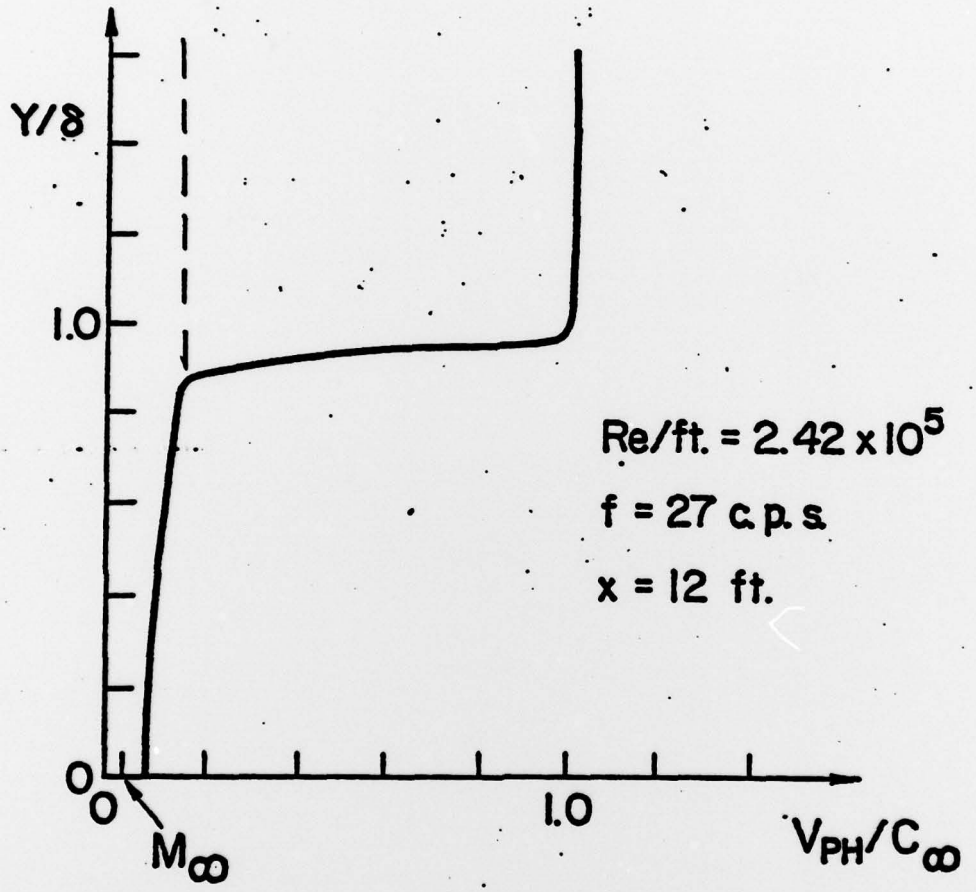

FIGURE 2. PHASE SPEED OF WAVES INSIDE BOUNDARY LAYER UNDER THE INFLUENCE OF ACOUSTICAL DISTURBANCE (DASHED LINE INDICATES PHASE SPEED WHEN FREESTREAM WAVE PROPAGATES AT U_∞)

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